Constrained coding for the deep space optical channel

Bruce Moision, Jon Hamkins

Throughputs for deep-space optical communication channels will be constrainted by the repetition rate of the laser. After the transmission of a pulse, there will be non-zero delay T_d during which the laser is recharging and may not transmit another pulse. This delay is significant for systems proposed for the deep-space channels. Letting T_s denote the symbol period, or slot duration, and assuming that T_d is divisible by T_s, the laser requires a delay of $d \stackrel{\text{def}}{=} T_d/T_s$ non-pulsed slots between pulsed slots. For an On-Off-Keyed system, this translates into a constraint that each 1 in the transmitted sequence must be separtated by at least d 0's. For clock synchronization it is reasonable to place an additional constraint that there is a maximum of k 0's, or non-pulsed slots, between 1's. We refer to this joint constraint on the maximum and minimum allowable time between pulses as a (d, k)-constraint.

(d, k) constraints also appear in magnetic and optical storage channels. The theory and application of modulation codes satisfying the constraint are well known. However, the d parameter for storage channels is typically on the order of $0--2$, while for the optical channel it is on the order of $64--512$. This significantly changes the problem.

One method of operating within a (d, k) constraint is to use M-ary Pulse Position Modulation (M-PPM). In M-PPM, a block of $log_2(M)$ bits modulates a frame of M slots which is followed by a dead time of d slots during which no information is transmitted. M-PPM is attractive due to its simplicity and moderate throughput, measured in user bits per slot.

In this paper, we demonstrate a class of low-complexity modulation codes satisfying the (d, k) constraint that offer throughput gains over M-PPM on the order of $10 - 15\%$, which translate into SNR gains of $.4 - .6$ dB.

The codes are are constructed as variable-rate, although the encoders and decoders are operated at a fixed rate. We illustrate a novel low-complexity variable-out-degree trellis for maximum-likelihood decoding of the codes (assuming an AWGN channel) and show that the variable-out-degree trellis may be used for extracting soft-output from the decoder. Lifting the constraint on having a fixed-outdegree permits lower complexity encoding and decoding.

We compare the modulation codes to M-PPM in terms of probability of bit error, throughput, slot and frame synchronization, and complexity. We also compare cost and performance of integrating the two systems with an outer error-correcting code.

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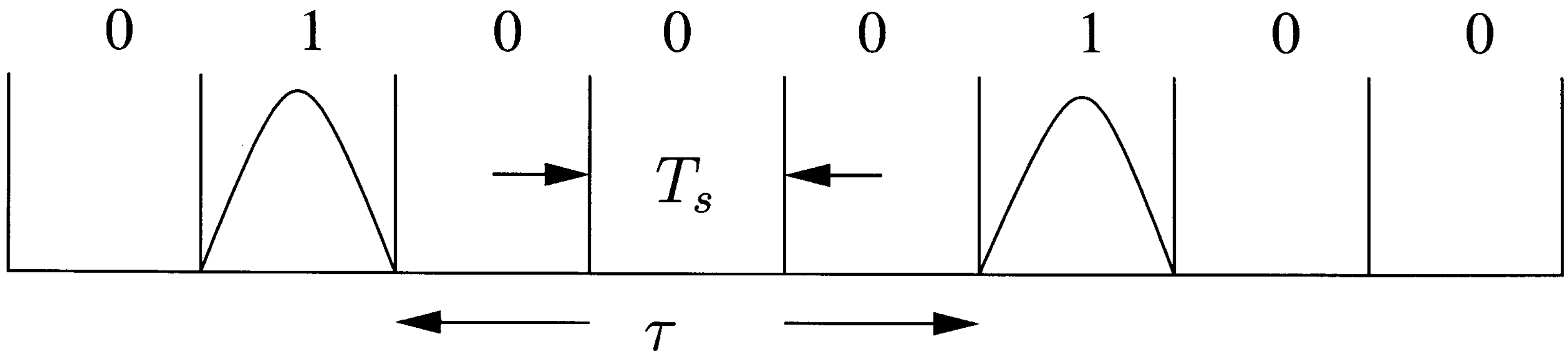
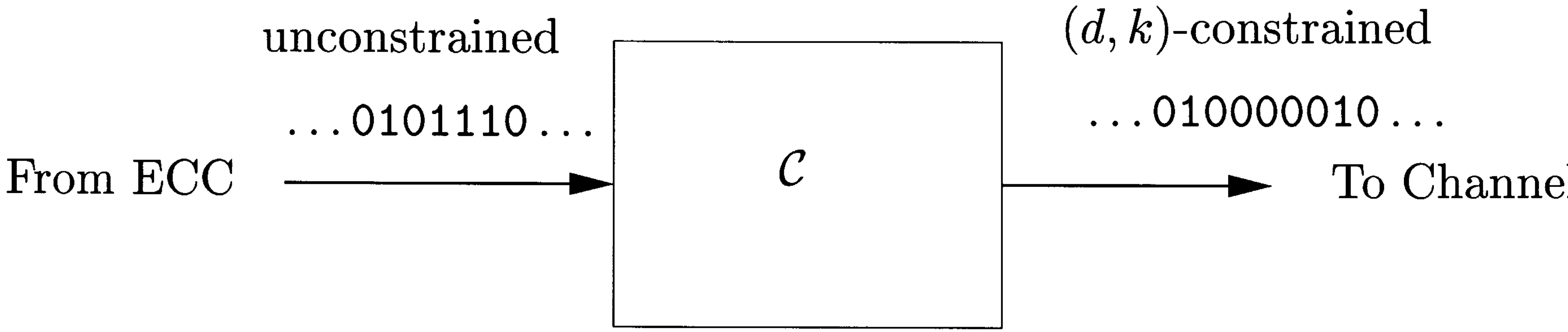
Dead-time constraint



$$T_d \leq \tau \leq T_k$$

- The optical channel is constrained on-off keying, where there is at least T_d and at most T_k seconds between pulsed slots. With on-off-keying, this translates into the constraint that 1's be separated by at least $d = (T_d/T_s)$ but no more than $k = (T_k/T_s)$ 0's.
- Refer to this as a (d, k)-constraint.

Constrained Code

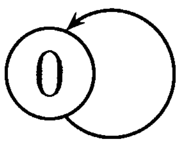


Constrained Code

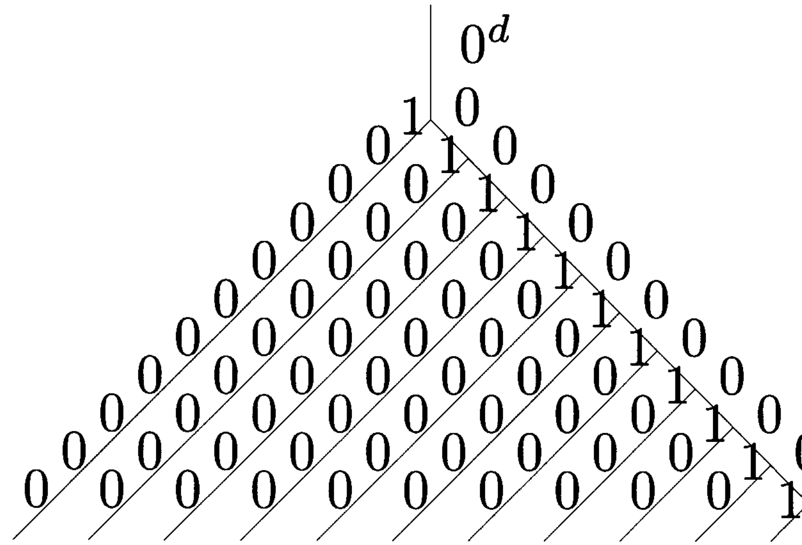
$$\mathcal{C} : \{0,1\}^p \to \{0,1\}^q$$

- What are the achievable rates, $R_{\mathcal{C}} = p/q$, of such a code?
- What are the tradeoffs? E.g. complexity/throughput/transmitted energy/performance?
- What is the performance in a larger coding scheme?

Example: Pulse-Position-Modulation (PPM) with deadtime



$$10^{M-1}0^d$$
$$010^{M-2}0^d$$
$$\vdots$$
$$0^{M-1}10^d$$



$$R_{\text{PPM}}(d,M) = \frac{1}{T_s}\frac{\log_2(M)}{M+d} \text{ bits/s}$$

Choosing M to maximize the rate,

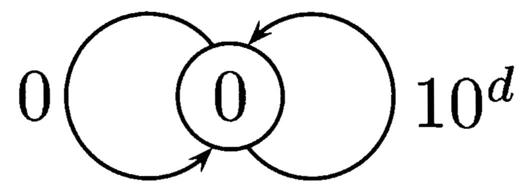
$$R_{\text{PPM}}(d) = \frac{1}{T_s\ln(2)}\frac{W(d/e)}{d} \text{ bits/s}$$

where $W(z)$ is the *productlog* function which gives the solution for w in $z = we^w$.

Achievable Rates

Describe allowable sequences as paths on a labelled graph.

Consider rates relative to (d, ∞), and take k as a design parameter.



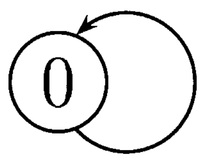
Capacity,

$$C(d) \stackrel{\text{def}}{=} \lim_{n \to \infty} \frac{1}{n} \log |\text{words of length } n \text{ in the } (d, \infty) \text{ system}|$$

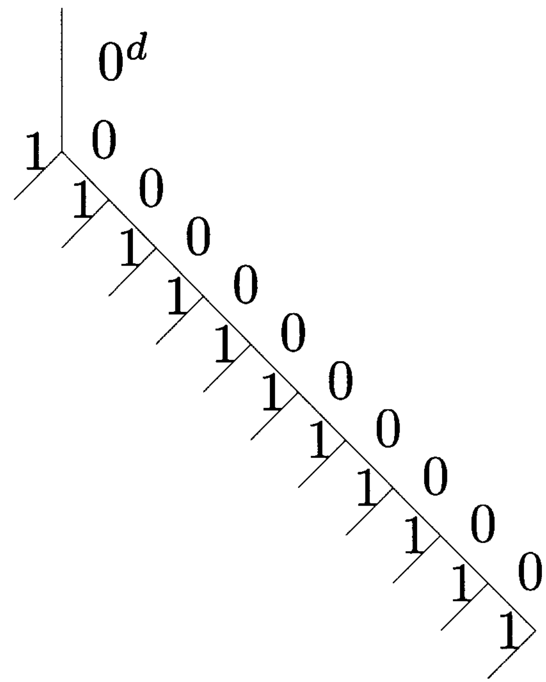
upper bounds achievable rates. For large d, we have *[Shannon, 48],[Khandekar, McEliece, 99]*.

$$C(d) \approx \frac{1}{T_s \ln(2)} \frac{W(d+1)}{d+1} \text{bits/s}$$

Truncated-Pulse-Position-Modulation (TPPM)



$$\begin{array}{c} 10^d \\ 010^d \\ \vdots \\ 0^{M-1}10^d \end{array}$$

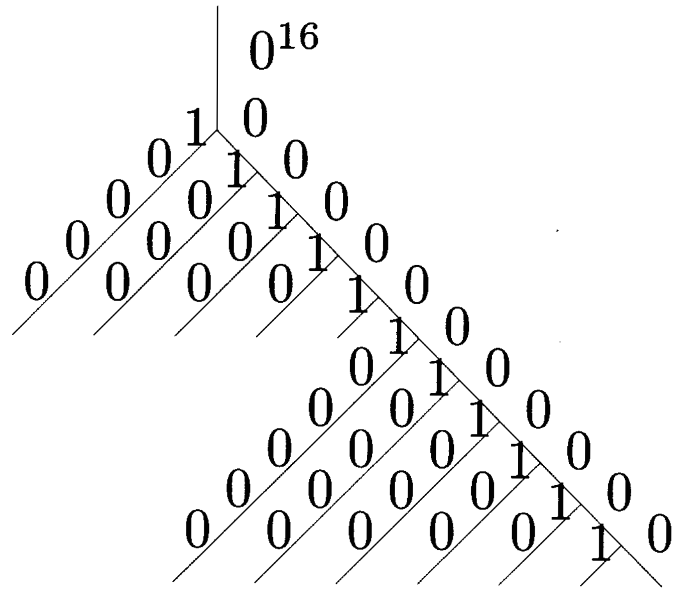


TPPM has a maximum *average* rate

$$R_{\mathrm{TPPM}}(d) = \frac{2}{T_s \ln(2)} \frac{W(\frac{2d+1}{e})}{2d+1} \text{ bits/s}$$

$R_{\mathrm{TPPM}}(d) > R_{\mathrm{PPM}}(d)$, hence $R_{\mathrm{TPPM}}(d)/C(d) \rightarrow_{d\rightarrow\infty} 1$. However, variable rate mapping leads to implementation problems.

STPPM, $d = 16$

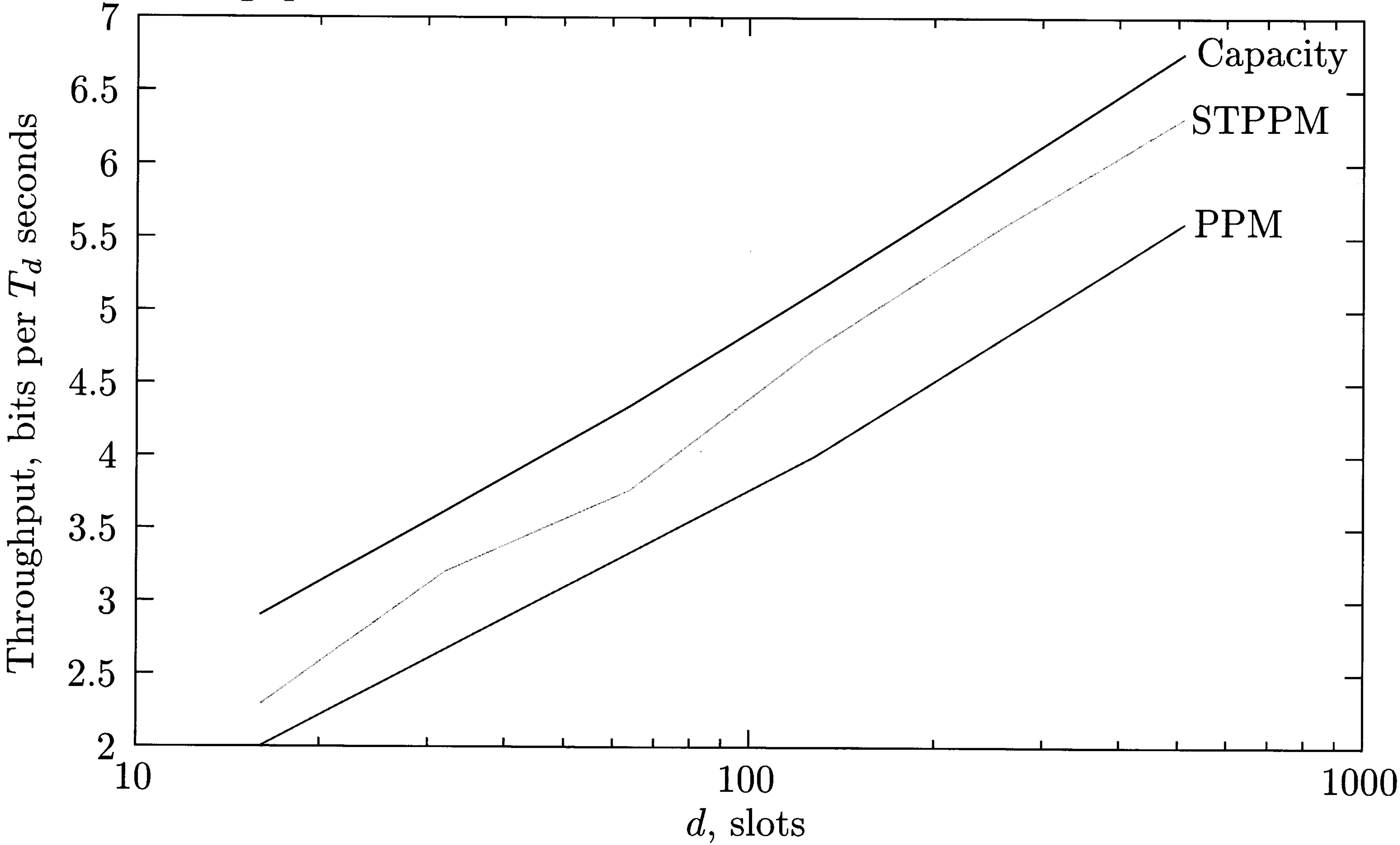


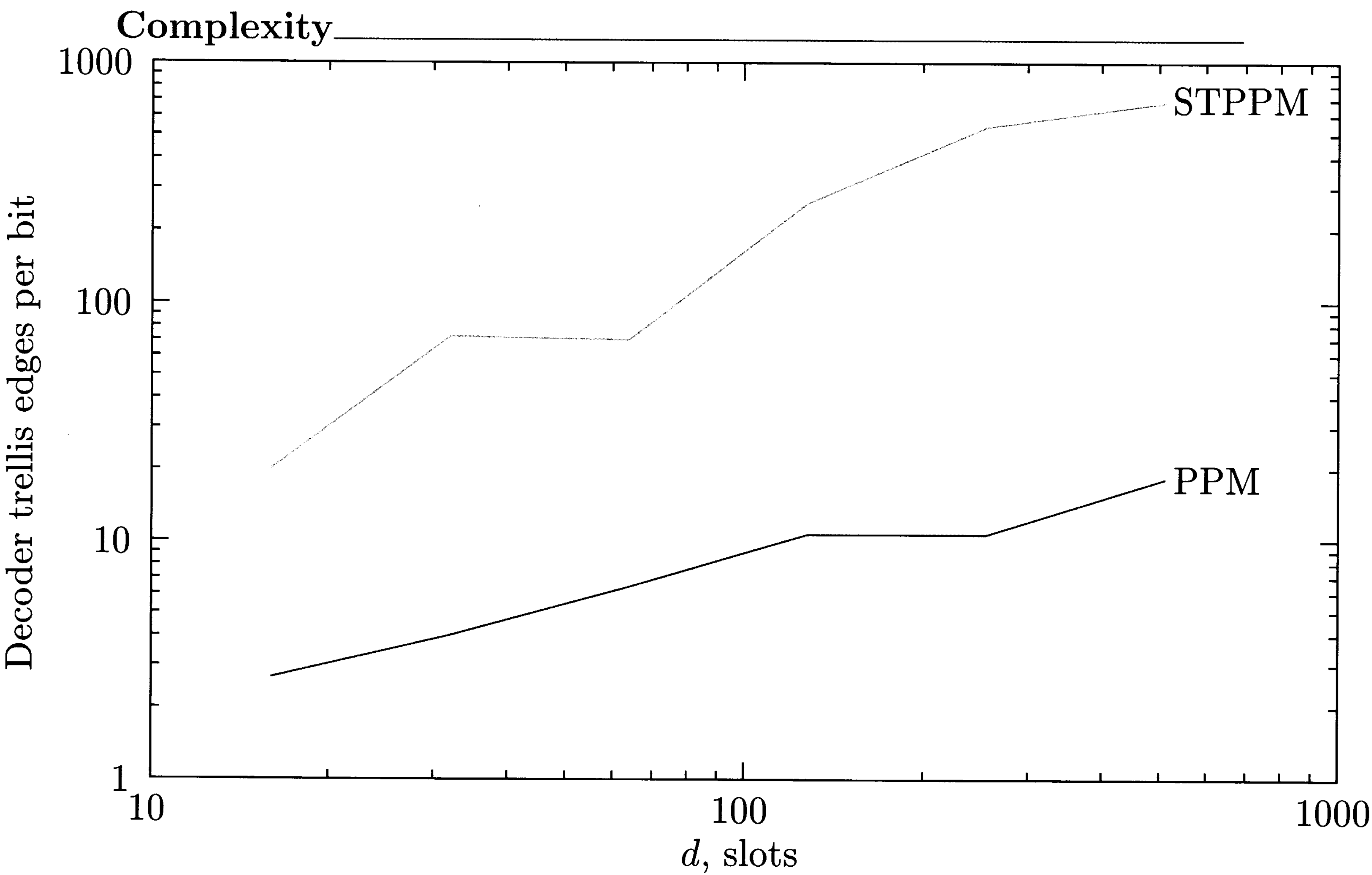
- Allow variable-length codewords, but constrain mapping to be synchronous, i.e.,

$$\mathcal{C} : \{0,1\}^{mp} \rightarrow \{0,1\}^{mq}, m = 1, 2, \ldots$$

hence rate is fixed, p/q. In example, mappings are $3/21, 4/28$, but may be implemented at a fixed rate $1/7$.

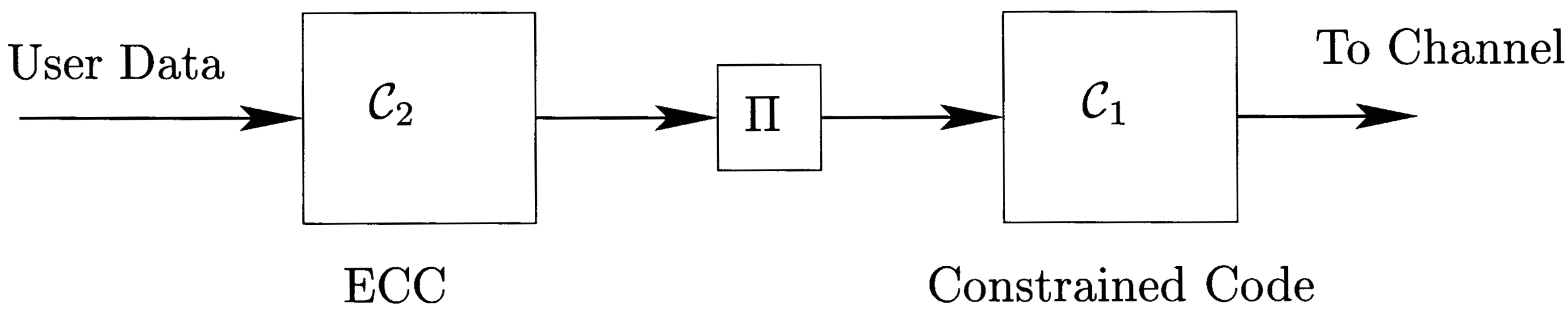
Throughput


Capacity
STPPM
PPM
Throughput, bits per T_d seconds
d, slots
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
10
100
1000



Complexity
1000
100
10
1
10
100
1000
Decoder trellis edges per bit
d, slots
STPPM
PPM

Concatenating the constrained code



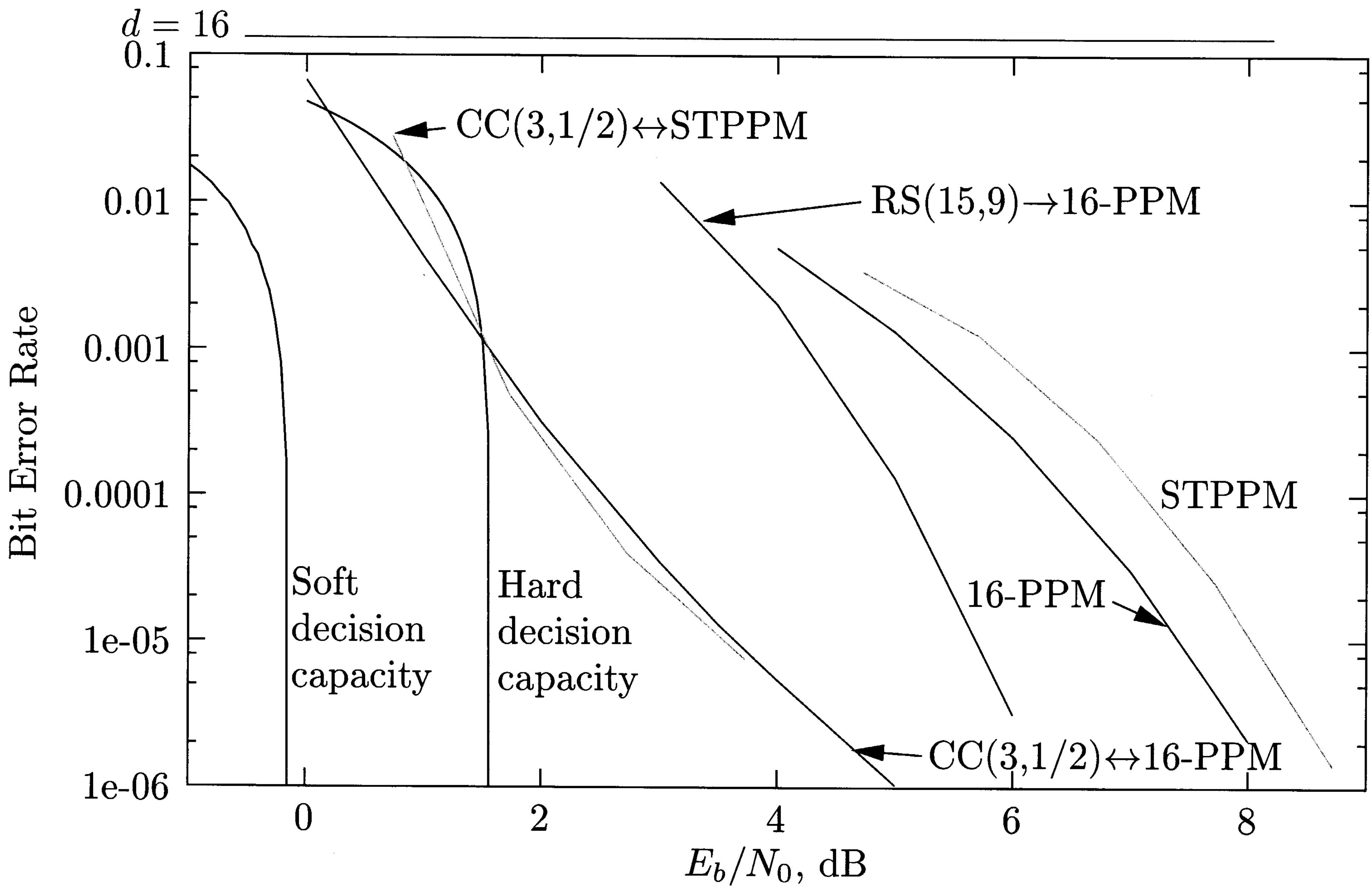
- Constrained code will be concatenated with an outer Error Correcting Code (ECC).
- Baseline is Reed Solomon concatenated with PPM ($RS(M-1,k) \leftarrow$MPPM).
- Other orders of concatenation, e.g. those considered for magnetic or optical storage, are inappropriate here.

Prior Work

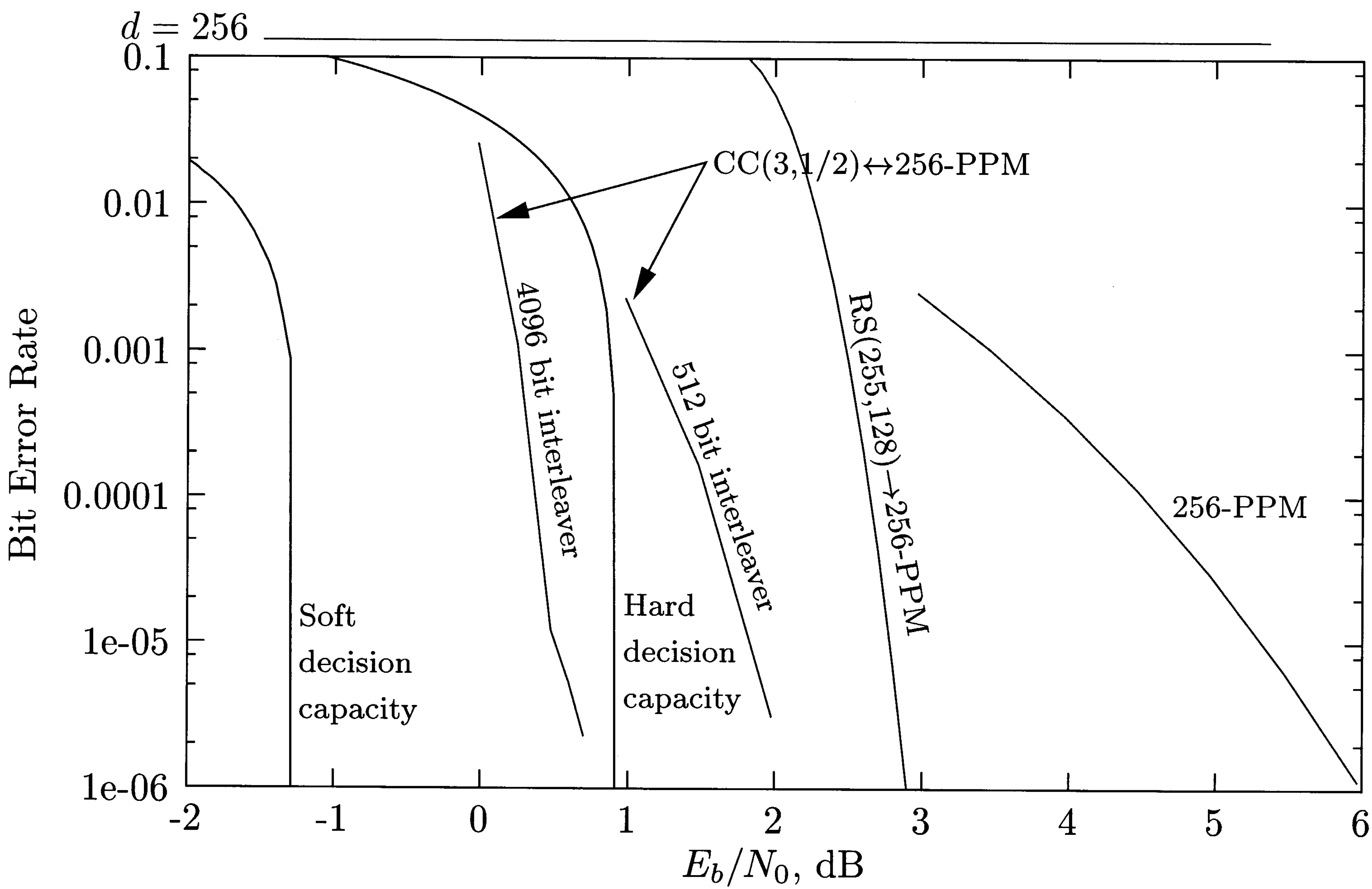
- PCCC$\leftarrow$PPM *[Hamkins, 99]* on AWGN, Webb, Webb+Gaussian channel models.
- PCCC$\leftrightarrow$PPM *[Peleg, Shamai, 00]* Included PPM in iterations on *discrete-time memoryless rayleigh fading channel.* Illustrated performance 1–2 dB from capacity. PPM introduced to yield distribution close to capacity achieving.

Proposed system

- We illustrate that the system $CC(3,1/2)$ $\leftrightarrow$PPM, or $CC(3,1/2)$ $\leftrightarrow$STPPM, where CC$(3,1/2)$ is a 4-state convolutional code, provides substantial gains over RS$(M-1,k)$ $\leftarrow$PPM, moderate gains over PCCC$\leftarrow$PPM, and small losses relative to PCCC$\leftrightarrow$PPM.



d = 16
Bit Error Rate
0.1
0.01
0.001
0.0001
1e-05
1e-06
0
2
4
6
8
Eb/N0, dB
CC(3,1/2)↔STPPM
RS(15,9)→16-PPM
STPPM
16-PPM
CC(3,1/2)↔16-PPM
Soft
decision
capacity
Hard
decision
capacity



$d = 256$
Bit Error Rate
0.1
0.01
0.001
0.0001
1e-05
1e-06
-2
-1
0
1
2
3
4
5
6
E_b/N_0, dB
CC(3,1/2)↔256-PPM
4096 bit interleaver
512 bit interleaver
RS(255,128)→256-PPM
256-PPM
Soft decision capacity
Hard decision capacity

Conclusions

- Trade-offs of complexity for throughput in replacing PPM.
- Clear gains of the serially concatenated, iteratively decoded schemes relative to baseline RS$\rightarrow$PPM.
- Low complexity serial concatenation $CC \leftrightarrow PPM$ performs well.